Marshallian demands exercise

Consider a consumer with preferences represented by the following utility function:

$$u(x, y) = 4x^{1/2} + 2y^{1/2}$$

This consumer has an income m and the prices of the goods are p_x and p_y, respectively. Find the Marshallian demands $x(m, p_x, p_y)$ and $y(m, p_x, p_y)$.

Solution

The consumer's Lagrangian is:

$$L = 4x^{1/2} + 2y^{1/2} - \lambda[p_x x + p_y y - m]$$

First order conditions for interior solutions:

$$\frac{\partial L}{\partial x} = 0 \Rightarrow 2x^{-1/2} = \lambda p_x$$
$$\frac{\partial L}{\partial y} = 0 \Rightarrow y^{-1/2} = \lambda p_y$$
$$\frac{\partial L}{\partial \lambda} = 0 \Rightarrow p_x x + p_y y = m$$

Optimality condition and we express y in terms of x:

$$MRS = \frac{p_x}{p_y} \Rightarrow \frac{2x^{-1/2}}{y^{-1/2}} = \frac{p_x}{p_y} \Rightarrow \left(\frac{y}{x}\right)^{1/2} = \frac{1}{2}\frac{p_x}{p_y} \Rightarrow y = \frac{1}{4}\left(\frac{p_x}{p_y}\right)^2 x$$

Substitute into the budget constraint:

$$p_x x + p_y y = m$$
$$p_x x + p_y \frac{1}{4}\left(\frac{p_x}{p_y}\right)^2 x = m$$
$$x\left(p_x + \frac{1}{4}\frac{p_x^2}{p_y}\right) = m$$
$$x\frac{1}{4}p_x^2\left(\frac{4}{p_x} + \frac{1}{p_y}\right) = m$$

Solving for x we get:

$$x(m, p_x, p_y) = \frac{4m}{p_x^2\,(4/p_x + 1/p_y)}$$

Inserting this into the y function, we get the other Marshallian demand:

$$y(m, p_x, p_y) = \frac{m}{p_y^2(4/p_x + 1/p_y)}$$